Harmonic Analysis Errors in Calculating Dipole, Quadrupole, and Sextupole Magnets using POISSON

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Introduction

The computer program POISSON was used to calculate the dipole, quadrupole, and sextupole magnets of the 6 GeV electron storage ring. A trinagular mesh must first be generated by LATTICE. The triangle size is varied over the "universe" at the discretion of the user. This note describes a series of test calculations that were made to help the user decide on the size of the mesh to reduce the harmonic field calculation errors. A conformal transformation of a multipole magnet into a dipole reduces these errors.

Dipole Magnet Calculations

A triangular mesh used to calculate a "perfect" dipole magnet is shown in Fig. 1. Both the physical (x-y) and logical (K-L) mesh coordinates are shown. The lower boundary of this "universe" is a flux normal boundary and can be considered the mid-plane of the magnet. The top boundary is also a flux normal boundary and can be considered as an infinite permeable pole tip. The left boundary is a flux line of vector potential 0.0 G-cm and the right boundary is also a flux line of vector potential 140,000 G-cm. Since the distance between these boundaries is 14 cm, the flux density in the universe will be uniformly 10000 gauss. A mesh 8 units high by 29 units wide was used. The total number of mesh points is (8+2) (29+2) or 310 mesh points. This includes the four phantom mesh lines surrounding that shown.

The harmonics are calculated by integrating the vector potential on a circular arc and doing a Fourier analysis of it. Hence, the program requests an integration radius, RINT, a starting angle, ANGLZ, a change in angle, ANGLE, and a normalization radius, RNORM. The number of terms to calculate, NTERM, and the number of equidistant points on the arc of circle, NPTC must also be specified. The integration radius and normalization radius were both 3.0 and 19 points were used on the circular arc from 0 to 180 degrees. These

19 mesh points are shown circled in Fig. 1. Table I gives the results of the calculation. All harmonics have units of gauss. The maximum error for a mesh this size is less than .05 gauss in 10000 gauss at a radius of 3 cm!

A smaller mesh size might be possible, but with this size, 0.5 by 0.5 cm, the mesh is not too distorted at RINT when 0.17 cm by 1.5 cm shims are attached to the pole tip at the sides. Equal weight triangles were used.

Quadrupole Magnet Calculations

It can be shown [1] that the pole shape for a perfect p-pole magnet satisfies equation (1).

$$r^{p/2} \sin (p/2)\theta = R_B^{\ p/2} \tag{1}$$

Likewise the coil shape satisfies equation (2).

$$r^{p/2} \cos (p/2)\theta = R_c^{\ p/2} \tag{2}$$

R_B is the distance to the pole and R_c the distance to the coil. In rectangular coordinates for a quadrupole magnet, p = 4, these become:

$$xy = R_B^2/2 \text{ (pole shape)} \tag{3}$$

and

$$x^2 - y^2 = R_c^2 \text{ (coil shape)} \tag{4}$$

A perfect quadrupole is shown in Fig. 2. As in the case of the dipole, the lower and upper boundaries are flux normal boundaries. The left side is a flux line at 45 degrees and the right side is a flux line of vector potential A_1 where

$$A_1 = \int_o^{R_c} B_y dx = B' \frac{R_c^2}{2} = (1000)\ \frac{G}{cm}\ \frac{(6.945)^2}{2}\ cm^2 = 24116.51 \text{ G-cm.}$$

It is best to distribute the triangles uniformly along the x axis and the 45 degree line, since the field varies linearly. This makes the change of the flux density the same across each triangle and makes the errors equal. POISSON assumes that the vector potential varies linearly across each triangle. This assumption conflicts with the quadrupole field which varies linearly with radius. This effect is illustrated in Fig. 3.

The distribution of mesh points along the pole tip can be found by solving equations 3 and 4 simultaneously for a fixed R_B and for the 15 values of R_c which are the x coordinates of the mesh points on the x axis. Similarly, the distribution of mesh points along the coil can be found by solving equations 3 and 4 simultaneously for a fixed R_c and for the 8 values of R_B along the 45 degree line. This method of distribution has been used to calculate the harmonics for four different mesh sizes. The results are given in Table II. A flux plot is shown in Fig. 4.

Using the same number of mesh points, 170, as were used in the dipole case results in field errors of 15 gauss in 3000 or 0.5 percent. Doubling the mesh in each direction results in 527 points and reduces the field errors to 3.2 gauss or 0.1 percent. Again, doubling the mesh for a total of 1829 points gives 1.9 gauss error or 0.06 percent. With 6785 mesh points, the error is 0.3 gauss or 0.01 percent. These results clearly demonstrate the conflict between the basic assumption of field uniformity in POISSON and the linear field of a quadrupole magnet. A method to circumvent this problem is described in the last section.

Sextupole Magnet Calculations

In rectangular coordinates for a sextupole magnet, p = 6, equations 1 and 2 become:

$$3x^2y - y^3 = R_B^3 \text{ (pole shape)} \tag{5}$$

$$x^3 - 3y^2x = R_c^3 \text{ (coil shape)} \tag{6}$$

A perfect sextupole is shown in Fig. 5. The lower and upper boundaries are flux normal boundaries. The left side is a flux line at 30 degrees of vector potential zero. The right side is a flux line of vector potential A_2 where

$$A_2 = \int_o^{R_c} B_y dx = B'' \frac{R_c^3}{3} = \frac{(100)}{3} (7.2569)^3 = 12738.9 \text{ G-cm.}$$

To distribute the mesh points along the x axis so that the change in field between successive points is the same requires that

$$x_N = (N/NPTS)^{1/2} (R_c)$$

where N is the nth point and NPTS is the total number of points along the x axis. A similar distribution can be made along the 30 degree line. Using these x_N values as R_c in equation 2, the two equations, 1 and 2, can be solved simultaneously to find the nth point on the pole tip. The points on the right boundary can be found in a similar way using the R_N as R_B.

The results of the calculations for two mesh sizes is shown in Table III. Using 432 mesh points results in errors of 1.5 gauss out of 3025 gauss or 0.05 percent. With 1364 mesh points, the error is 0.6 gauss or 0.02 percent. A flux plot is shown in Fig. 6.

Conformal Transformation

It has been shown[2] that higher pole magnets can be transformed into a dipole magnet by the transformation:

$$W = \frac{Z^{p/2}}{(\frac{p}{2}) R_o^{(p/2)-1}} \qquad (7)$$

where

W = u + iv

Z = x + iy

p = number of poles

R_o = the magnet bore radius.

for p = 4, a quadrupole magnet, these become

$$u = \frac{x^2 - y^2}{2R_o} \qquad v = \frac{xy}{R_o} \tag{8}$$

The pole tip and coil shown in Fig. 7a of a quadrupole magnet is transformed by equations 8 into the pole tip and coil of a dipole magnet of Fig. 7b. The program LATTICE transforms the current from the x-y plane into the u-v plane. The user must first transform the x-y geometry into the u-v geometry and use this as input to LATTICE. The total current is the same in both planes[2].

POISSON transforms the permeability and prints out the fields, gradients, etc. in both the x-y and u-v planes for the air points. The fields in the steel are printed out as they would be in the x-y plane. The stored energy is the same in both planes. A discussion of the advantages, limitations and drawbacks is given in reference 2 and will not be repeated here.

References

1. Robert J. Lari and Gerald J. Bellendir, unpublished report dated June 1, 1967.

2. K. Halbach, "Application of Conformal Mapping to Evaluation and Design of Magnets Containing Iron with Nonlinear B(H) Characteristics," Instruments and Methods, 64 (1968) pp 278-284.

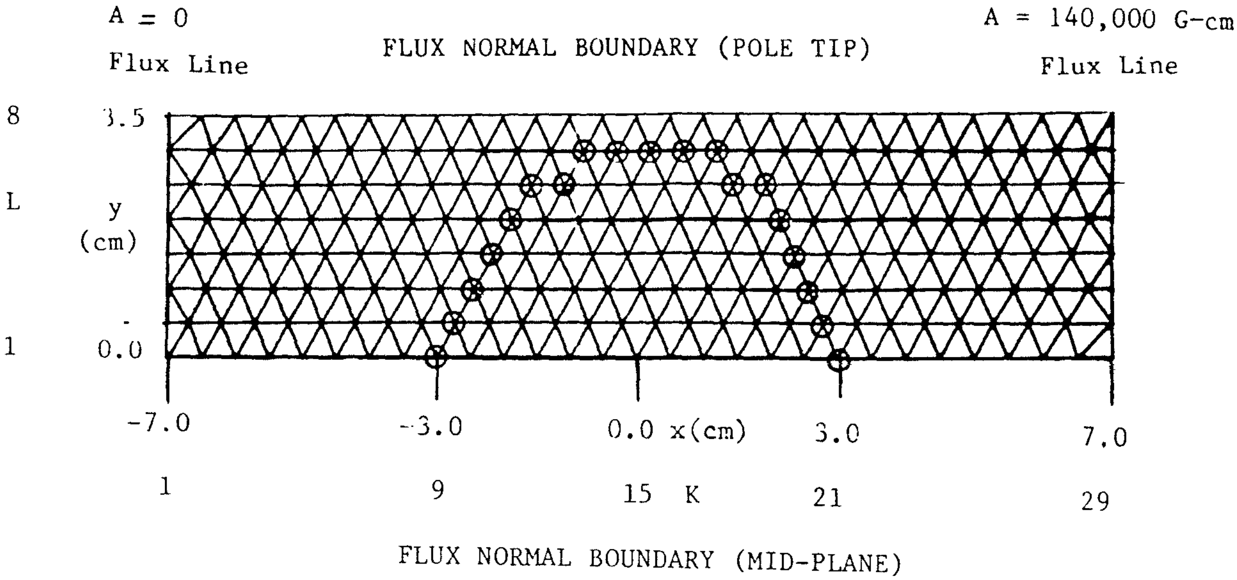
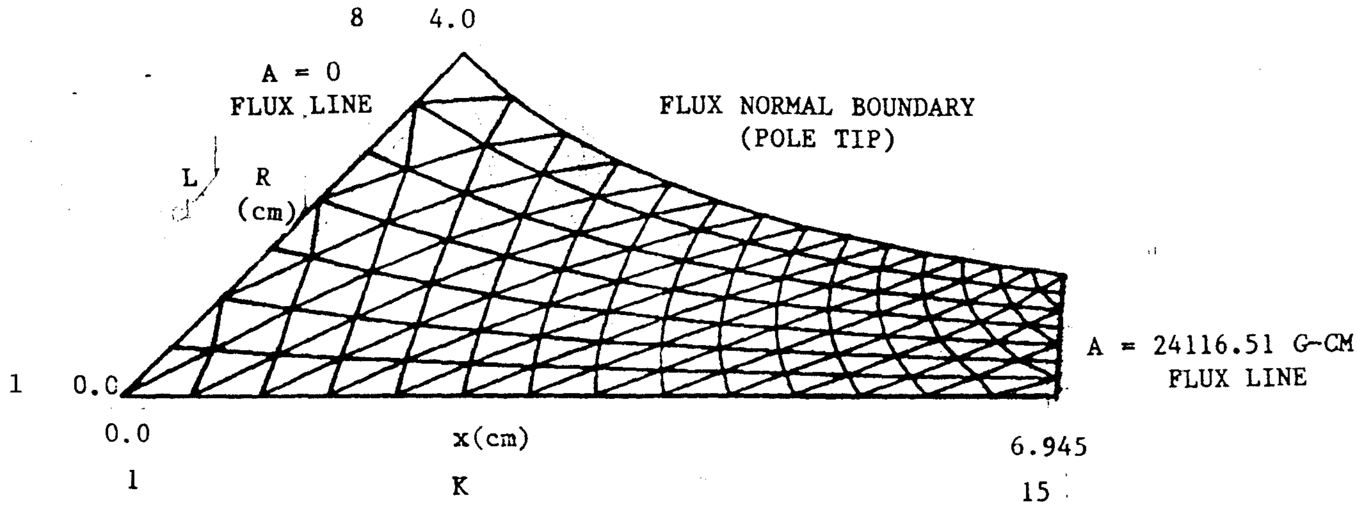


FIGURE 1. PERFECT DIPOLE MAGNET

n	B_n (gauss)
1	10000.000
2	.002
3	- .032
4	.034
5	- .027
6	.042
7	- .047

TABLE I



FLUX NORMAL BOUNDARY (MID-PLANE)

FIGURE 2. PERFECT QUADRUPOLE MAGNET

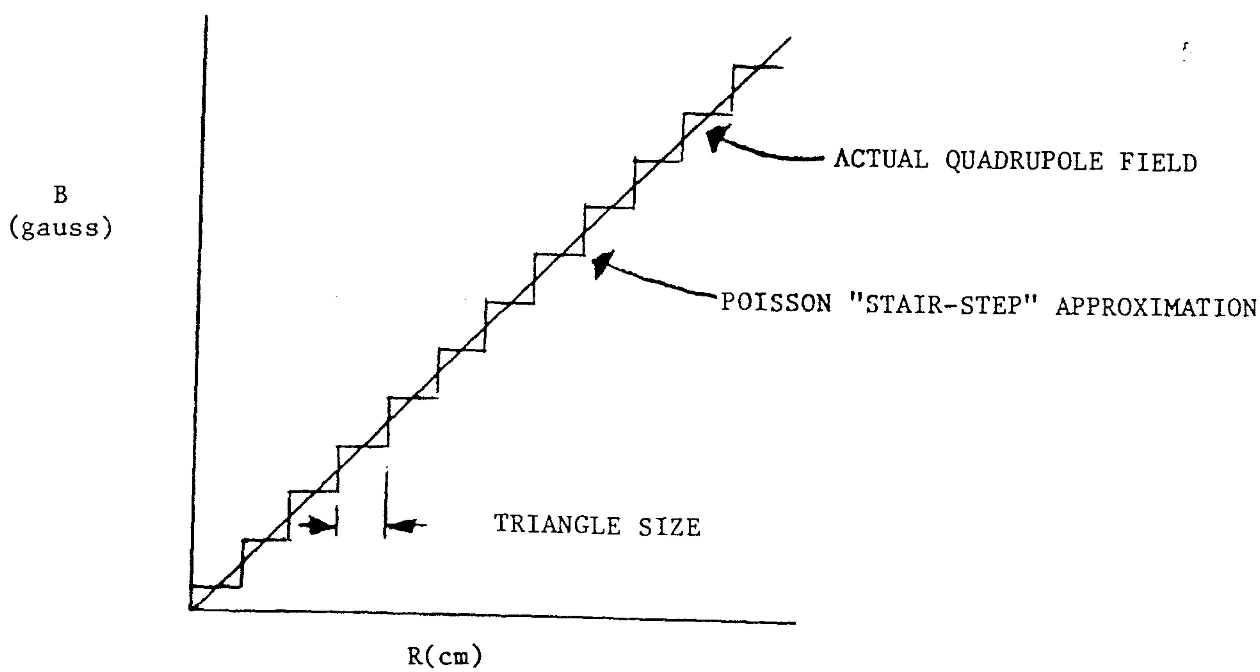


FIGURE 3. POISSON APPROXIMATION TO A QUADRUPOLE FIELD

n	B_n (gauss)(at RNORM = 3.0 cm)			
2	2985.7	2996.8	2998.1	2999.7
6	14.54	2.45	1.20	0.12
10	−6.00	−0.96	−0.48	−0.21
14	3.67	0.35	−0.03	−0.08
18	−2.29	−0.32	0.05	−0.12
22	0.47	0.15	−0.02	−0.27
26	0.36	0.01	0.02	−0.04
NO. OF MESH PTS.	170	527	1829	6785

TABLE II. A PERFECT QUADRUPOLE MAGNET HARMONIC CALCULATIONS

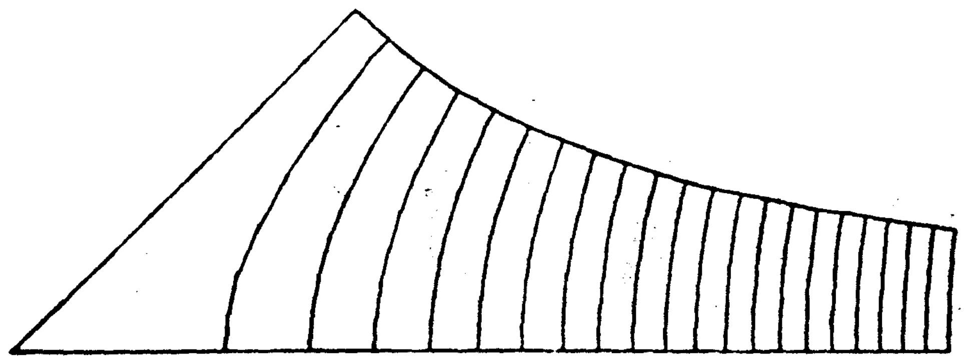


FIGURE 4. PERFECT QUADRUPOLE FLUX LINES

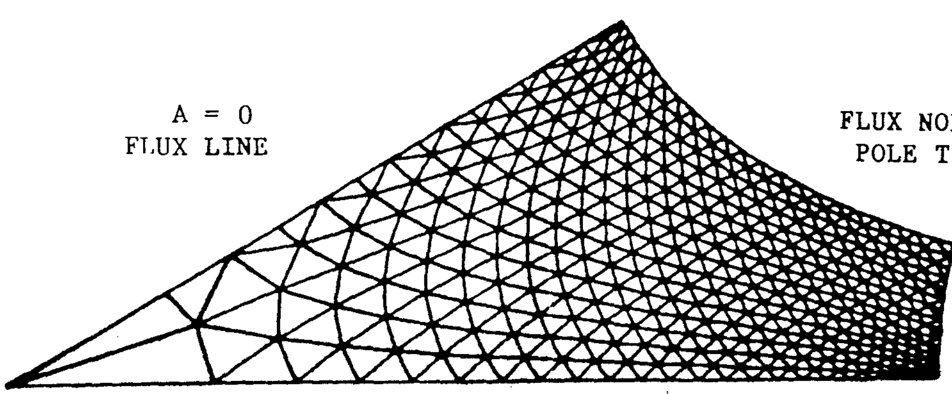


FIGURE 5. PERFECT SEXTUPOLE MAGNET

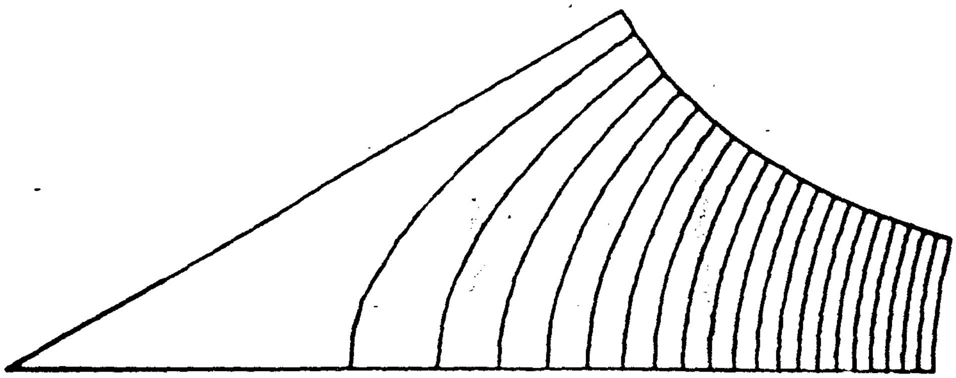


FIGURE 6. PERFECT SEXTUPOLE MAGNET FLUX LINES

n	(RNORM = 5.5 cm) B_n (gauss)	
3	3023.4	3024.5
9	-0.28	-0.02
15	-1.59	-0.66
21	-0.40	-0.02
No. of Mesh Pts.	432	1364

TABLE III. A PERFECT SEXTUPOLE HARMONIC CALCULATIONS

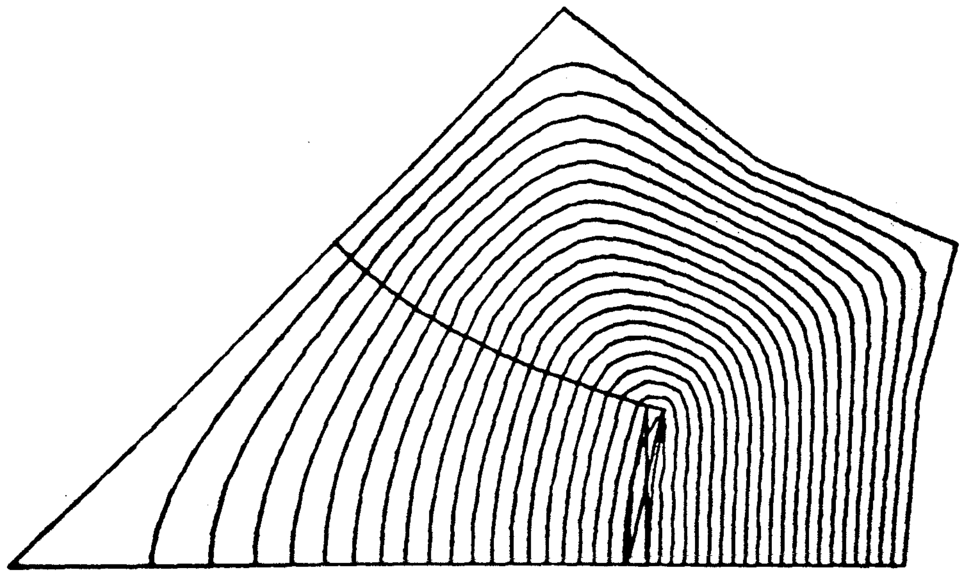


FIGURE 7a. QUADRUPOLE IN THE x-y PLANE

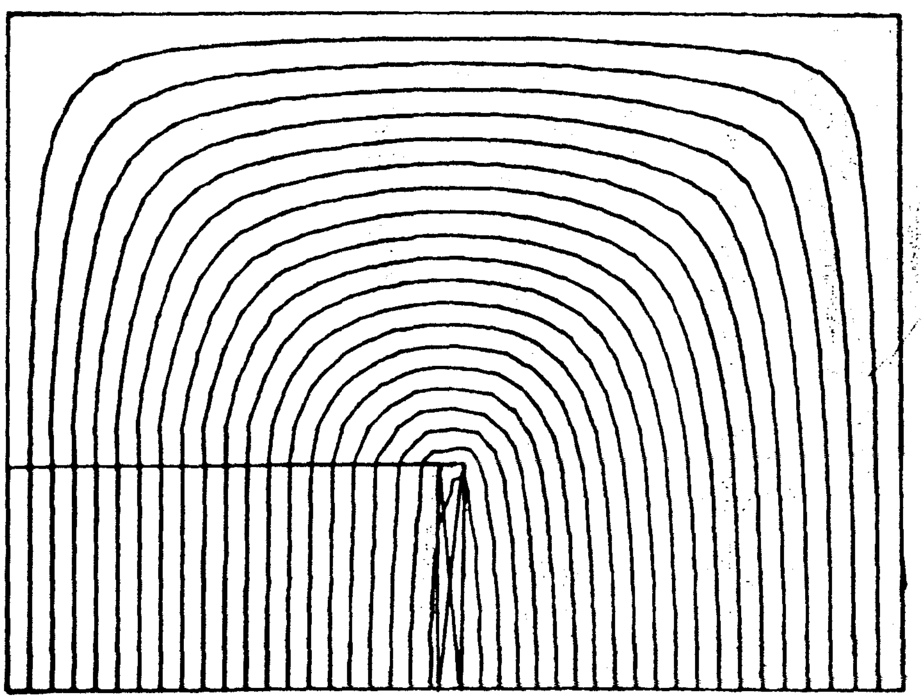


FIGURE 7b. QUADRUPOLE TRANSFORMED INTO A
DIPOLE MAGNET IN THE u-v PLANE